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Biological Cybernetics

Optical acceleration cancellation: a viable interception strategy?

L. A. Rozendaal, A. J. (Knoek) van Soest

Faculty of Human Movement Sciences, Vrije Universiteit Amsterdam,
Van der Boechorststraat 9, 1081 BT Amsterdam, The Netherlands

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Abstract. Interception of fly balls requires active locomotion toward the point where catching can take place; as a result, the visual information guiding interception is affected by the catcher's own movement. The only interception theory currently available for a catcher standing in the plane of motion of the ball is Optical Acceleration Cancellation (OAC); in this strategy, the pseudo-optical variable "optical acceleration" (OA), if nonzero, specifies how the catcher should adjust his current velocity. We formulate a precise implementation of OAC where the catcher strives to maintain OA zero at all times and analyze its implications in terms of the catcher's interception behavior for different ball trajectories under air-friction-free, low-friction, and friction-dominated conditions. We conclude that the point in the ball trajectory where first visual contact (FVC) takes place determines to a large extent the ensuing interception behavior of the catcher. Conventional trajectories (FVC slightly above eye level, ball coming toward the catcher) result in fast acceleration to a constant velocity and successful interception. Trajectories with FVC below eye level typically result in unsatisfactory behavior of the catcher, who runs away from rather than toward the point of interception. In addition, ball trajectories are identified for which the OA equals zero even though the catcher is not on an interception course.

We find that two different formulations of OAC in the literature ("get rid of OA" and "choose acceleration direction based on the sign of OA") actually represent different strategies. Results of this study show that the first formulation is effective for a limited class of ball trajectories only. Regarding the second formulation, which was not analyzed in detail, we argue that it cannot result in a generally adequate strategy either and conclude that variables other than OA are indispensable for successful interception.

Correspondence to: L. A. Rozendaal
(e-mail: l_rozendaal@fbw.vu.nl,
Tel.: +31-20-4448481, Fax: +31-20-4448529)

1 Introduction

Catching fly balls (e.g., in baseball) is an interception task in which locomotion toward the point of interception is a prerequisite for success. During locomotion phase, toward the interception point perceptual and motor processes are mutually coupled: the flow of visual information in some way specifies the catcher's movement toward the ball, while at the same time the catcher's movement affects the flow of visual information. As a result, it is not straightforward how the catcher may base his running behavior on the available visual information.

Several studies have assessed the question what visual information is necessary to identify the path of the ball or the point of interception (Todd 1981; Brancazio 1984; Saxberg 1987a,b); however, in these studies the observer remains stationary. The only actual interception theory originates from Chapman (1968), who considers the situation where the ball to be caught heads directly toward the catcher and follows a parabolic path. Chapman observes that a catcher moving at a constant velocity could determine from a single optical variable if this velocity will lead him to interception. This variable h is the tangent of the angle of the line from eye to ball with the horizontal plane or, in a different formulation, the position of the ball on a virtual vertical projection plane moving with the catcher at unit distance from the eye (Fig. 1A). For the constant catcher velocity leading to interception, the second derivative of h or "optical acceleration" equals zero up to the moment of interception (Fig. 1C). When the constant velocity is too low to result in interception, the optical acceleration will be negative (Fig. 1B), whereas a positive optical acceleration indicates that the current velocity is too high (Fig. 1D). It has been found that during successful interception of fly balls the catcher settles on an approximately constant velocity for which the reconstructed optical acceleration is approximately zero (Michaels and Oudejans 1992; McLeod and Dienes 1993). For balls not heading straight toward the catcher a Linear Optical Trajectory (LOT) theory has been proposed, which maintains that the catcher should run

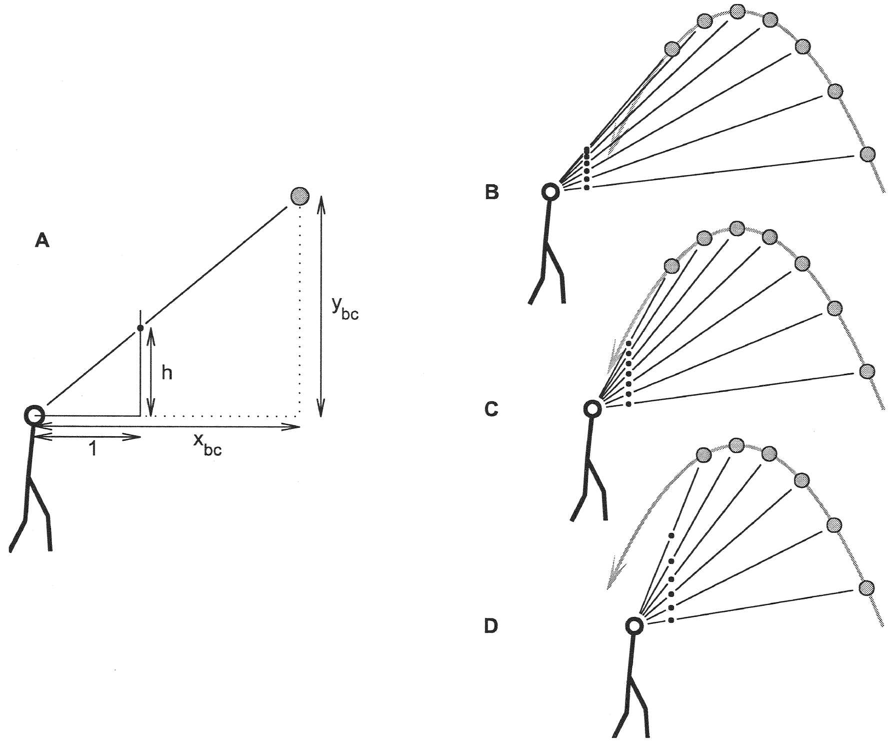

Fig. 1. A Interpretation of $h = y_{bc}/x_{bc}$ as the projection of the ball on a virtual screen at unit distance from the catcher's head. **B** For parabolic ball trajectories, if the catcher is behind the interception point, he experiences a negative optical acceleration (projection points shown are equidistant in time). **C** A catcher standing at the interception point sees the projection of the ball on the virtual projection plane move with a constant velocity, i.e., with zero optical acceleration. **D** If the catcher is in front of the interception point, the optical acceleration is positive. When the catcher moves at a constant velocity, the trajectory of the ball relative to him remains parabolic; hence the figures also represent the situation of a catcher who moves toward the interception point with a constant velocity that is too low (**B**), correct (**C**), or too high (**D**)

such that he sees the ball moving along a straight line, curving neither upwards nor downwards (McBeath et al. 1995). However, LOT only couples the catcher's lateral movement to his movement toward the ball, for which he needs an additional strategy. Some experimental data appear to be consistent with LOT (Shaffer and McBeath 2002), but in other experiments the catcher's behavior would seem to preclude LOT as a general strategy (McLeod et al. 2001). Another method to cope with the catcher's lateral movement was proposed by Tresilian (1995).

By extension of the observations of Chapman, loosely defined Optical Acceleration Cancellation (OAC) interception strategies have been proposed. McLeod and Dienes (1993) argue that keeping the optical acceleration zero will keep the angle of the ball with the horizon between 0 and 90 degrees, thus ensuring interception. The catcher would wait for half a second before he starts to run, accelerate until he reaches a velocity where the optical acceleration is zero, and then modulate his speed up to the point of catching, maintaining the optical acceleration close to zero. Michaels and Oudejans (1992) suggest that the catcher should "get rid of" the optical acceleration and state that locomoting in such a way as to zero out the optical acceleration will yield an intersection of the ball and eye trajectories. To achieve this the catcher should accelerate forward when the optical acceleration is negative and accelerate backward when it is positive.

A complicating factor in these intuitive generalizations is the tacit assumption that the optical acceleration is determined solely by the catcher's position and velocity. However, the catcher's acceleration also directly affects the optical acceleration he observes. Oudejans et al. (1996) mention this fact in the appendix to their article, but the implications for the interception strategy have not been investigated yet.

To assess if OAC is a viable interception strategy and compatible with experimentally observed velocity profiles, we reformulate it with explicit reference to the catcher's acceleration. As the OA depends directly on the physical acceleration, in principle it is possible for the catcher to cancel out OA just by selecting the appropriate acceleration. For the sake of argument we assume that the catcher is capable of controlling his acceleration to such an extent that he may keep the OA zero at all times. Thus, we do not consider the constraints following from the dynamics of the effector system but focus solely on the behavior of a catcher who ideally adheres to what the term OAC implies: cancellation of the OA. This control strategy, which we will denote as exact OAC, results in a differential equation specifying the catcher's behavior for a given ball trajectory. Despite the nonlinear nature of this differential equation, an analytical solution can be obtained for arbitrary ball trajectories. Using the differential equation and its solution for parabolic, low-friction, and friction-dominated ball trajectories, we investigate under what conditions the exact OAC dynamical law leads to catcher behavior that is qualitatively consistent with experimental observations.

2 Exact OAC: dynamical law and its solution

The optical acceleration is the second time derivative of the optical variable $h(t)$ defined by:

$$h(t) = \frac{y_b(t) - y_c(t)}{x_b(t) - x_c(t)} = \frac{y_{bc}(t)}{x_{bc}(t)} \tag{1}$$

Here $[x_b\, y_b]^T$ and $[x_c\, y_c]^T$ are the position of ball and catcher in an inertial coordinate system, respectively, and $[x_{bc}\, y_{bc}]^T$ is the ball position relative to the catcher. The OA experienced by the catcher follows by differentiating Eq. 1:

$$\ddot{h} = \frac{(x_{bc}\ddot{y}_{bc} - \ddot{x}_{bc}y_{bc})x_{bc} - 2(x_{bc}\dot{y}_{bc} - \dot{x}_{bc}y_{bc})\dot{x}_{bc}}{x_{bc}^3} \tag{2}$$

Throughout our analysis, it will be assumed that the catcher's eye remains at a constant level, set at zero for convenience: $y_c(t) = 0$. Equation 2 indicates that the OA depends not only on the catcher's position and velocity, but also on his acceleration. According to our rigorously defined OAC strategy, the catcher will aim to keep the OA zero at all times. Setting $\ddot{h} = 0$ in Eq. 2, we obtain the dynamical law for exact OAC:

$$\ddot{x}_c = \ddot{x}_b - \frac{x_{bc}}{y_b}\ddot{y}_b - 2\dot{x}_{bc}\left(\frac{\dot{x}_{bc}}{x_{bc}} - \frac{\dot{y}_b}{y_b}\right) \tag{3}$$

Equation 3 specifies the catcher's acceleration that cancels out $\ddot{h}$, given the current motion of the ball $(x_b, y_b, \dot{x}_b, \dot{y}_b, \ddot{x}_b, \ddot{y}_b)$ and the catcher's current state $(x_c, \dot{x}_c)$. Alternatively, it may be interpreted as the differential equation that specifies the movement of a catcher who from the moment of first visual contact keeps the optical acceleration $\ddot{h}$ exactly at zero. This ideal OAC catcher follows a trajectory that is specified dynamically but uniquely by the ball trajectory and his own state when he first sees the ball. Such a "catcher" may be thought of as a lumped mass and a propelling force that can instantaneously assume the required magnitude to generate the acceleration prescribed by Eq. 3. It should be stressed that the second-order nature of the OAC differential equation does not arise from the catcher's inertia; rather, it results from the fact that the catcher's acceleration directly affects the OA he observes, i.e., from the second-order nature of the OAC strategy.

The exact OAC dynamical law (Eq. 3) contains two singularities, which occur if the ball is just above the catcher ($x_{bc} = 0$) or when it is at eye level ($y_b = 0$). From its appearance it would seem unlikely that the catcher trajectory $x_c(t)$ can be expressed explicitly in terms of the ball trajectory. However, an analytical solution is readily obtained from the definition of exact OAC itself. For the ideal OAC catcher the second derivative of $y_{bc}(t)/x_{bc}(t)$ equals zero at all times; consequently $y_{bc}(t)/x_{bc}(t)$ itself must be a linear function of time:

$$\frac{y_{bc}(t)}{x_{bc}(t)} = c_0 + c_1 t \tag{4}$$

The values of the constants c_0 and c_1 follow from the initial conditions at $t = 0$, which is the moment of first visual contact (FVC). With $[x_{bc,0}\; y_{b,0}]^T$ the initial ball position relative to the catcher and $[\dot{x}_{bc,0}\; \dot{y}_{b,0}]^T$ the relative ball velocity, after reordering it is found that:

$$\frac{x_{bc}(t)}{x_{bc,0}} = \frac{y_b(t)}{y_{b,0}}\frac{1}{1+kt} \quad \text{with} \quad k = \frac{\dot{y}_{b,0}}{y_{b,0}} - \frac{\dot{x}_{bc,0}}{x_{bc,0}} \tag{5}$$

As before, it has been assumed that $y_c(t) = 0$. The catcher trajectory $x_c(t)$ for any ball trajectory is obtained by substituting the appropriate $x_b(t), y_b(t)$. A remarkable consequence of the analytical solution is that whenever $y_b = 0$, $x_{bc} = 0$ as well, i.e., the ideal OAC catcher will always intercept the ball perfectly whenever it reaches eye level (with the exception of an occasional degenerate solution). The value of the coefficient k strongly affects the nature of the solution. In particular, when k is negative and sufficiently large, the hyperbolic term $1/(1+kt)$ may tend to infinity at some time before the first interception possibility, which implies that the catcher-ball distance will become unboundedly large. Such nonfinite solutions are consistent with the presence of singularities in the differential equation; how they relate to the finding that all interception possibilities will lead to exact interception is left to the reader's imagination for the moment.

In the following, we will investigate several air-friction regimes. When air friction is negligible, ball trajectories are parabolic. For limited air friction conditions the ball trajectory is governed by a set of coupled differential equations:

$$\begin{aligned} m\ddot{x}_b &= -\frac{1}{2}\rho C_d A_p \sqrt{\dot{x}_b^2 + \dot{y}_b^2}\; \dot{x}_b \\ m\ddot{y}_b &= -\frac{1}{2}\rho C_d A_p \sqrt{\dot{x}_b^2 + \dot{y}_b^2}\; \dot{y}_b - mg \end{aligned} \tag{6}$$

Parameter values used are $\rho = 1.23$ kg/m^3 (air density), $C_d = 0.45$ (drag coefficient for low velocities), $m = 0.453$ kg (mass of the ball), and $A_p = 0.039$ m^2 (frontal area of the ball); the latter two values describe a ball as used in soccer (de Mestre 1991). For this parameter set the relative-friction parameter $\rho C_d A_p/m$ equals 0.050. For comparison we mention the parameter values for a baseball: $m = 0.145$ kg, $A_p = 0.00428\;\; m^2, C_d = 0.2$ (high velocities) -0.5 (low velocities) (de Mestre 1991; Adair 1994), corresponding to a relative-friction parameter in the range 0.0073–0.018.

We determined low-friction ball trajectories using the partially analytic solution method of de Mestre (1991). In comparing the friction-free and low-friction cases, we use ball trajectories with the same initial position, interception position, and flight time; this implicitly specifies the initial ball velocity, which in the case of the low-friction simulations is determined by an iterative numerical procedure.

When we analyze the case where air friction is dominant, as for a badminton shuttle (for which the parameter combination $\rho C_d A_p/m$ is large), we consider only the part of the trajectory where the gravitational and friction forces are in equilibrium and the vertical ball position is a linear function of time. While admittedly a somewhat artificial situation, it may be easily obtained in a carefully designed experimental situation and, like the parabolic ball trajectories, it presents a useful and easily analyzable limit case of "natural" friction conditions.

In the interpretation of the results, the catcher is assumed to be at rest initially at the origin of the coordinate system ($x_{c,0} = 0, \dot{x}_{c,0} = 0$). This does not limit generality as the coordinate system used may always be regarded as the momentarily comoving reference frame of the catcher at $t = 0$ (by the principle of Galilean relativity). The catcher is assumed to be looking in the direction of the positive x-axis, and we assume that balls are visible to him when $x_{bc} > 0$.

3 Results for friction-free and low-friction trajectories

Ball trajectories used in interception research are generally of the type shown in Fig. 2: starting in front of the catcher and ending some distance ahead or behind him. For all trajectories in Fig. 2 first visual contact (FVC) is made when the ball is 0.5 m above eye level. The catcher velocity profiles show an initial acceleration phase followed by a period of approximately constant velocity. The period of acceleration is short and largely independent of the shape of the ball trajectory. In the presence of limited air friction, the catcher's velocity toward the interception point is not strictly constant, as the ball trajectory is not strictly parabolic. However, the differences with the friction-free case are slight. For this reason, in the following we will concentrate on friction-free parabolic ball trajectories, which allow rigorous analysis.

The point where the ball is first seen has a considerable impact on the catcher's behavior. A situation is considered in which as a result of experimental manipulation or due to physical obstacles the catcher does not see the ball until it has reached a certain height. Figure 3 shows the catcher's responses on balls thrown along a single trajectory but with varying FVC height. If visual contact occurs just above eye level, the catcher reaches a constant velocity after a short acceleration period as before. If the point of FVC lies higher, the velocity builds up considerably slower and is still increasing at the moment of interception. The point of FVC may also be below eye level; for example, in baseball (taking the point where the bat hits the ball as the point of FVC) and more markedly when an object is thrown to someone standing at an elevation (podium, scaffold, first floor). In this case the ideal OAC catcher will not run toward the ball but backwards with an ever-increasing velocity (the mathematical catcher then reappears at "plus infinity" and successfully completes interception). Figure 4 shows why this behavior is required for exact OAC. The relation between $y_{b,0}$ and the catcher's behavior can be interpreted from the differential Eq. 3. If $y_{b,0}$ is near zero, the terms in which the inverse of $y_{b,0}$ occurs are large and cause a substantial acceleration of the catcher, which in the present context leads him toward the ball when $y_{b,0} > 0$ and consequently away from the ball when $y_{b,0} < 0$. If $y_{b,0}$ is large, this "acceleration drive" is less markedly present, resulting in a much slower increase of the catcher's velocity (even though from a functional point of view a larger acceleration would be useful, as the time to interception is smaller).

For friction-free conditions, the boundaries between ball trajectories leading to finite and nonfinite catcher trajectories can be specified explicitly (Appendix 1). Determining factors are: the locations where the ball path passes eye level, the initial relative velocity of the ball (decreasing or increasing the ball-catcher distance), and whether the ball is above or below eye level at FVC. The results are summarized in Fig. 5. Balls that are seen

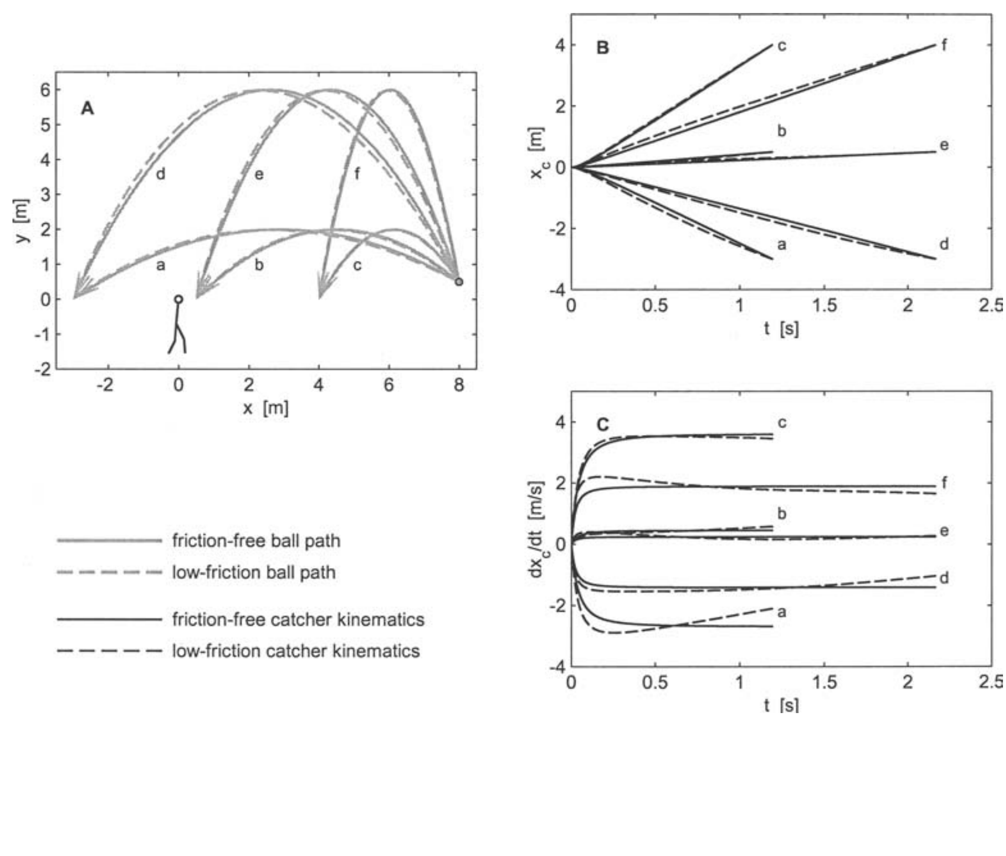

Fig. 2A–C. Interception of a ball following several friction-free and limited-friction trajectories according to exact OAC. All trajectories start at 0.5 m above eye level. **A** Ball trajectory. **B** Position of the catcher. **C** Velocity profile of the catcher. All ball trajectories shown lead to adequate catcher behavior

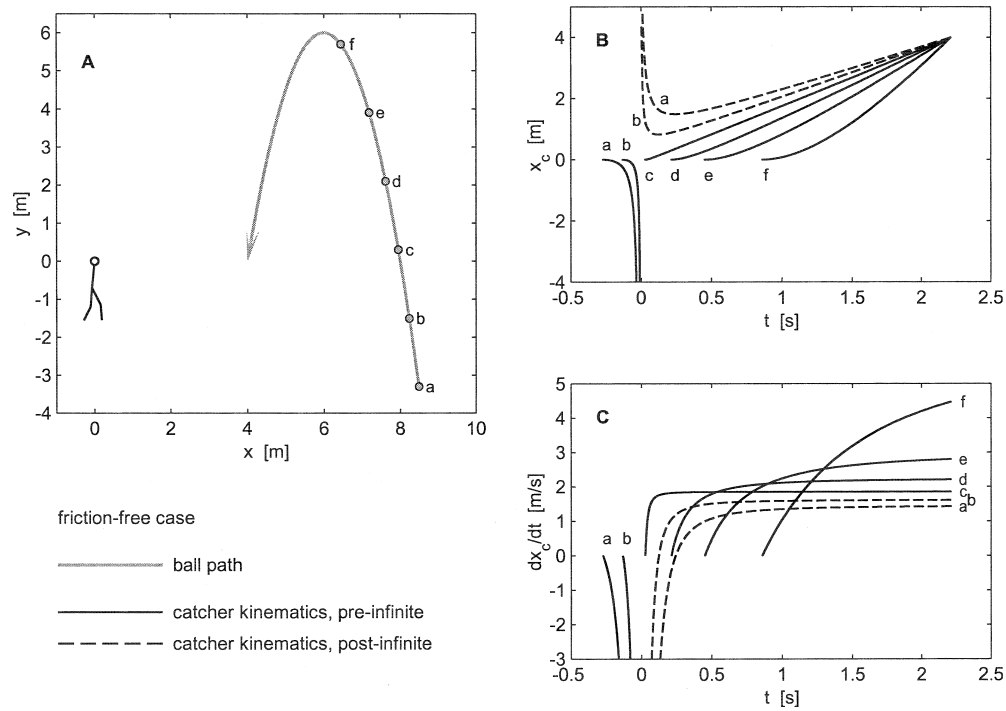

Fig. 3A–C. Interception of a ball on a friction-free trajectory (comparable to Fig. 2f) according to exact OAC, when visual information is occluded until the ball reaches a certain height. **A** Ball trajectory. **B** Position of the catcher. **C** Velocity profile of the catcher. If visual information starts when the ball is above eye level (from points $c/d/e/f$ onwards), interception occurs along a finite catcher trajectory. Velocity builds up slower as the point of first visual contact lies higher. If visual information starts below eye level (from points a/b onwards), the catcher behavior is nonfinite

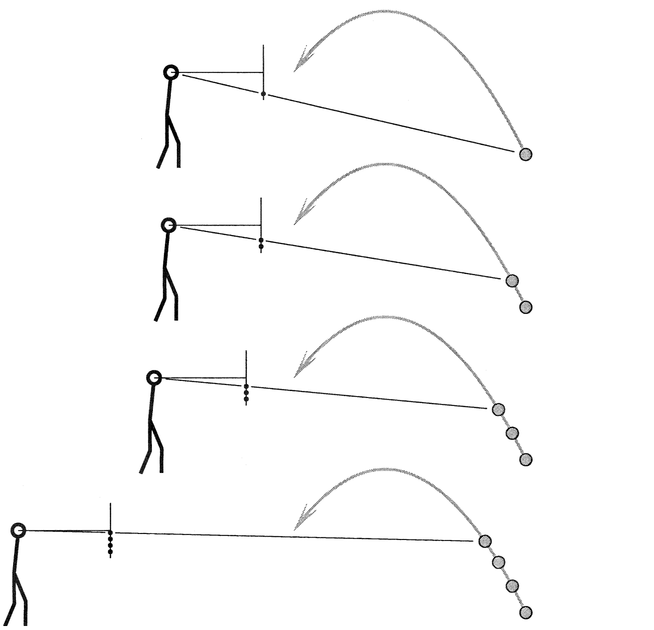

Fig. 4. When the ball approaches eye level from below, a large backward velocity is required to keep the velocity of projection constant (positions shown are equidistant in time)

first when above eye level are generally intercepted in a finite way (Fig. 5e–g). Only when the ball is thrown away from the catcher and the ball's parabolic path lies above the catcher's initial position the ensuing behavior will be nonfinite (Fig. 5h). The boundary between the finite and nonfinite trajectories is formed by balls thrown away from the catcher on a trajectory that appears to originate at the catcher's eye. These trajectories are the time reverse of ball trajectories ending at the catcher's position, and as such they also result in a zero OA (see Fig. 1C, reversing the order of the points). From an OAC point of view there is no need to start moving – and no clue in which way to proceed.

Balls starting below eye level pose more problems. In principle there are two interception possibilities: one while the ball is ascending and one while it is descending. When the ball is thrown away from the catcher (Fig. 5l), or when it is thrown toward him and both interception possibilities lie in front of him (Fig. 5i), his trajectory will become nonfinite even before the first interception possibility. When the ball passes over him (Fig. 5j), his trajectory will be finite up to the first interception possibility but become infinite before the second interception possibility. When both interception possibilities lie behind him (Fig. 5k), his trajectory is finite up to the second interception possibility; and as he is at the same position as the ball both times, it does not matter when he grabs it.

A real catcher shows a tendency toward constant velocity near interception (e.g., Michaels and Oudejans 1992), i.e., after the initiation of the movement the magnitude of the catcher's acceleration typically decreases with time. Hence catcher trajectories for which the magnitude of the acceleration continues to increase over time are not realistic. For parabolic ball trajectories resulting in finite catcher trajectories it can be shown that the increase or decrease of acceleration over time is completely determined by the sign of the coefficient k in Eq. 5, which in turn is determined by the state of the ball at FVC (Appendix 2). The decreasing-acceleration requirement is satisfied if the ball is first seen above eye level and the velocity vector is directed above the line from eye to ball or if the ball is first seen below eye level and then has a velocity

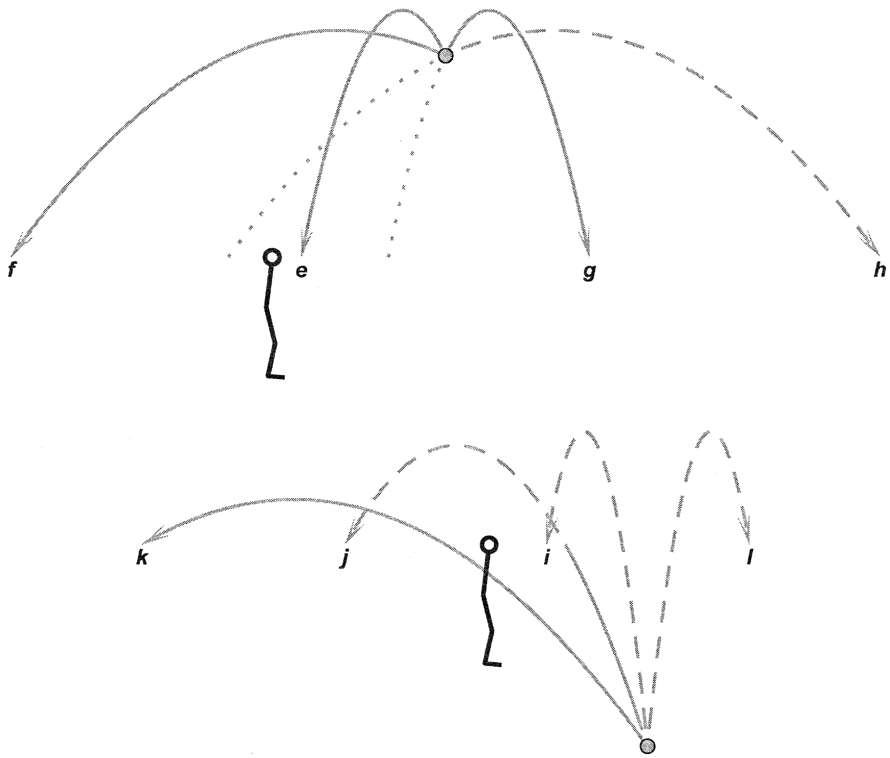

Fig. 5. The path of the ball relative to the initial position of the catcher determines whether or not the catcher's interception path will be finite. Ball path labels correspond to the analysis in Appendix 1 (categories $a/b/c/d$ do not yield interception opportunities and are not shown). *Drawn lines*: ball trajectories that are intercepted in a finite way ($e/f/g/k$); *broken lines*: ball trajectories that are intercepted in a nonfinite way ($h/i/l$); *combined drawn/broken line*: ball trajectories for which the catcher's trajectory is finite up to the ascending interception possibility but nonfinite between the ascending and descending interception possibility (j). *Dotted lines* are backward extensions of trajectory types g and h; for type g trajectories the catcher is "above" the ball path, for type h trajectories "below" it. For further description see text

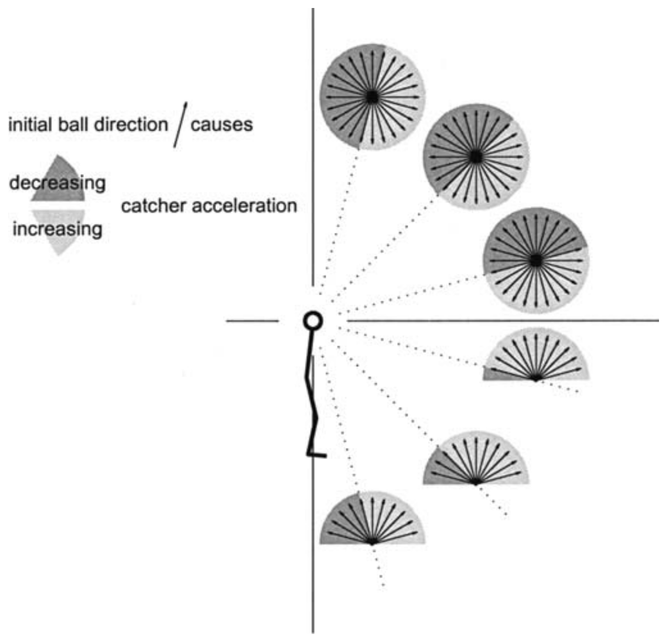

Fig. 6. The magnitude of the catcher's acceleration may decrease or increase over time; the latter is incompatible with actual catcher behavior. In exact OAC, the distinction between these two situations is determined by the throw direction of the ball (*arrows*) relative to the direction from the catcher's eye to the ball (*broken line*) at FVC. The catcher's acceleration decreases in magnitude only when the ball direction (*arrow*) points into a dark area. Note the qualitative difference for FVC above and below eye level. The results shown hold both for friction-free and for friction-dominated vertical ball trajectories

component below the line from eye to ball (Fig. 6). From the finite ball path categories in Fig. 5, the following are unrealistic based on the resulting acceleration profiles of the catcher: a subset of the trajectories of types e and g, type j, and a small subset of type k.

4 Results for friction-dominated vertical trajectories

In this section, we consider high-friction objects like a badminton shuttle falling down (which for simplicity we still denote as ball). When transient effects have subsided, gravitational and frictional forces are in equilibrium and the downward velocity of the ball is constant. We assume that FVC takes place at that point. A nonzero relative velocity $\dot{x}_{bc,0}$ may occur under these conditions because the catcher is moving at FVC or (somewhat more artificially) due to an air flow that gives the ball some constant horizontal velocity. For consistency with the friction-free case, we discuss the friction-dominated ball trajectories in terms of the latter situation.

Analysis of these trajectories is straightforward (Appendices 1 and 2); results are illustrated in Fig. 7. If initially the catcher-ball distance increases ($\dot{x}_{bc,0} > 0$), the catcher's trajectory will be nonfinite (Fig. 7d,e). If $\dot{x}_{bc,0}$ is slightly negative, the approach of the ball will be finite but the catcher's velocity profile quite unbalanced; initially the catcher hardly accelerates, while close to the moment of interception a very large acceleration is required in compensation (Fig. 7a,b). The minute initial acceleration results from the fact that very small catcher accelerations are sufficient to cancel out the observed optical acceleration until the ball approaches eye level. The boundary between finite and nonfinite behavior is formed by $\dot{x}_{bc,0} = 0$, e.g., the catcher is in rest initially and the ball falls straight down. In this case the catcher remains at rest (Fig. 7c); the ball moves at a constant

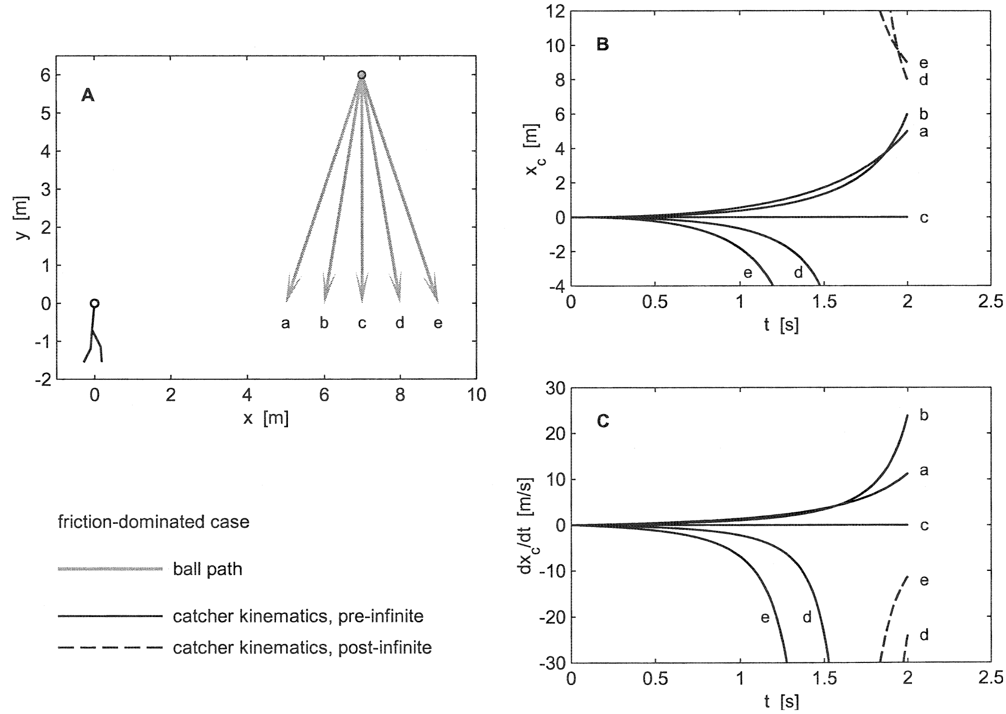

Fig. 7A–C. Interception of a ball falling downwards according to exact OAC under conditions where air friction is dominant. **A** Ball trajectory. Trajectories start with identical vertical velocities ($\dot{y}_{b,0} = -3$ m/s) and various horizontal ball-catcher velocity differences. **B** Position of the catcher. **C** Velocity profile of the catcher. The trajectories toward the catcher lead to successful interception, though with an unbalanced velocity profile (a/b). Trajectories vertically downward do not cause the catcher to move (c), whereas trajectories away from the catcher result in nonfinite behavior (d/e)

velocity over his virtual projection plane, and no movement is required to keep the optical acceleration zero.

5 Overview and Discussion

The main attractiveness of the Optical Acceleration Cancellation (OAC) strategy is its conceptual simplicity: one variable specifies whether the catcher's current constant velocity will lead him to interception. However, in its loosely defined form (Michaels and Oudejans 1992; McLeod and Dienes 1993), the strategy neglects the effect of the catcher's own acceleration on the OA he observes. Any change in velocity immediately influences the OA which is the supposed trigger for the change of velocity; it is not obvious how this will affect the ensuing interception behavior. For instance, when a small acceleration of the catcher toward the interception point is sufficient to cancel out the optical acceleration, no substantial velocity buildup may ensue.

We started out by formulating a rigorously defined OAC control scheme, which essentially specifies the way in which the catcher should move so as to keep the observed OA zero at all times. We showed that the behavior resulting from this scheme can be determined in closed form for arbitrary ball trajectories. For balls thrown toward the catcher and first seen when above eye level the exact OAC dynamical law predicts catcher behavior that is qualitatively correct: the catcher accelerates to a constant velocity and intercepts the ball. Quantitatively, it was found that the speed of velocity buildup depends strongly on the initial height of the ball, a prediction that can be tested experimentally. The dependence is not functional, however; if the ball is seen later in its trajectory, the initial acceleration of the catcher will be smaller, whereas the time to interception is shorter. The effect of incorporation of a realistic amount of air friction was found to be limited.

Other friction-free ball trajectories do not result in satisfactory interception behavior. For example, balls thrown away from the catcher in such a way that they appear to originate from the catcher's eye cause the catcher to observe a zero OA while remaining at rest, just like balls aimed directly at the catcher's eye position. The same occurs in the friction-dominated case for objects dropping straight down in front of the catcher. In these situations, the catcher will not be instigated to change his state of motion, even though this is clearly required for successful interception. Note that this problem is not specific to our current interpretation of OAC; the existence of such "devious" ball trajectories makes it unlikely that any control law based on OA alone will be able to describe general catching behavior.

When FVC takes place below eye level, the majority of friction-free ball trajectories cause the catcher to run backwards toward infinity at ever-increasing speed; we illustrated that this behavior is indeed required to keep the optical acceleration zero. The same happens for balls thrown away from the catcher on a flight path passing over the catcher's head. It would be interesting to test experimentally if subjects experience particular difficulty when trying to catch objects under these conditions.

Michaels and Oudejans (1992) proposed that (1) the catcher should get rid of the optical acceleration and (2) he could achieve this by accelerating forwards on an observed negative OA and backwards on a positive OA. However, from our analysis we conclude that (1) and (2) above in fact are different strategies. As an example, imagine a catcher at rest looking at a ball that will land in front of him (under friction-free conditions). Independent of whether the ball is above or below eye level at FVC, this situation will result in a negative OA. It follows that, when following strategy 2, the direction of the catcher's initial acceleration is independent of the ball height at FVC: he will accelerate forwards. In contrast, we showed in this study that when following strategy 1, typically the catcher will accelerate forwards when the ball is above eye level but backwards when the ball is below eye level. For FVC below eye level, in strategy 1 the catcher keeps OA zero but thereby removes himself from the ball, while in strategy 2 he moves toward the interception location but in doing so increases the magnitude of the observed OA. Only strategy 1 could be called optical acceleration cancellation. Strategy 2 should be named differently, e.g., Optical Acceleration Feedback (OAF), but it might result in more generally successful interception behavior than OAC.

For the purpose of discussion we introduce the following basic control law as a possible implementation of OAF:

$$\ddot{x}_c = -\kappa\ddot{h} \tag{7}$$

Now the catcher's acceleration is specified by the observed OA, which itself depends on the catcher's acceleration. By substitution, a differential equation is obtained for the catcher's interception behavior resembling Eq. 3 but dependent on the value of the gain κ. For ball-game-type trajectories and FVC above eye level, exact OAC and OAF are similar when κ is positive and sufficiently large; but in general the schemes may result in quite different catcher behavior. The control law (Eq. 7) can easily be generalized to more realistic dynamical models of a catcher, in which case the feedback term $\kappa\ddot{h}$ does not directly specify the catcher's acceleration but forms an acceleration cue that after sensory processing and filtering by the catcher's musculoskeletal system effectuates an acceleration. Such more realistic models should also include sensory thresholds on the perception of the optical acceleration. Tresilian (1995) performed a model study in which optical acceleration was the control variable for interception. The catcher's behavior was characterized by a McRuer-Krendel operator with physical limitations. For conventional ball trajectories, the model was found to perform well for a considerable range of initial positions.

Although OAF may have a larger range of applicability than OAC, it will not be a fully general interception strategy. In part this is due to the existence of the zero-OA ball trajectories that do not hit the catcher, as identified earlier. Furthermore, under "normal" (ball-game-type) conditions the gain κ should be positive, a negative optical acceleration instigating forward acceleration of the catcher; however, preliminary analysis suggests that for a set of less current ball trajectories κ should be negative (Appendix 3). Hence the sign of κ depends on the nature of the ball trajectory, and more information is required than OA alone.

Both OAC and OAF directly couple perception to action: sensory information is used instantaneously and deterministically to determine the catcher's behavior. Therefore, the trajectory of the ball uniquely specifies the trajectory of the catcher. The same ball trajectory will result in the same catcher trajectory, no matter how often it is repeated. Movement is not based on functional demands but on an input-output rule that sometimes may yield functional responses and sometimes not. Following OAC, an outfielder who just wiped some dust from his eye and now sees the ball while it is already in mid-air does not start off double quick to make up for the lost time; instead, as a result of the changed optical stimulus his initial acceleration will be slower than when he had seen the ball from the beginning.

Perception and action are likely to be related in a less rigid way. Sensory information gathered over a time interval can be integrated with knowledge of the physical environment (an internal model) to obtain an estimate of the time and distance to interception. Such internal models are a widespread concept in neuroscience and cognitive science (Kawato 1999). This knowledge need not be represented in the form of the differential equations we would use to describe it formally; the laws need not be exact, and their implications do not even have to be known accurately on a cognitive level, as indeed they do not appear to be (Hecht and Bertamini 2000). The only requirement is that available information be used to determine a prediction of the interception variables. The trajectory identification approaches of Todd (1981) and Saxberg (1987a,b) are along this line of thought. Research comparing gravity and no-gravity conditions suggests that subjects use an internal model with a fixed value of the gravitational acceleration (McIntyre et al. 2001). Based on this information a trajectory can be planned, taking into account (subconsciously and heuristically) such diverse subgoals as obstacle avoidance, effort minimization, and interception robustness. In the course of the interception, the incoming sensory information may be used to improve the initial estimates and to update the trajectory accordingly. This separation of perception and action is clearly suggested by the observation that some experienced catchers prefer to run to the point of interception and wait there until the ball arrives, even turning their back to the ball during the running phase (e.g., comments on McBeath et al. 1995 in: Science 1995, vol. 268: 1682–1683). Such a conceptualization of interception strategy is sometimes criticized on the grounds that it requires unrealistic amounts of neural processing. In our view, the amount of neural processing is not necessarily much larger than that required in the OAC strategy, which is simple in concept but in its neural implementation actually requires summation of angular information from visual and proprioceptive sources, whose second derivative of the tangent must be determined.

The above does not necessarily mean that OA is wholly without use in interception. It is clear that over the course of the movement, the time available to make effective compensatory actions gradually decreases. Optimizing subgoals becomes less relevant, whereas the primary goal of the movement – the interception itself – gains in importance. Parallel to this, the method of information processing is likely to shift from internal models and quasioptimal planning to direct feedback of the sensory signal, i.e., to instantaneous perception-action coupling. For this final phase, OAC/OAF might be candidate strategies; the visual accelerations during this final phase are so high that it is expected that the limited sensitivity of the eye for visual acceleration (Brouwer et al. 2002) will not cause a problem. However, discriminating experimentally between strategies close to the moment of interception will not be easy, as their predictions probably will not differ very much. Differentiating between direct coupling or decoupling of perception and action appears to be more feasible, e.g., by investigating to what extent catcher behavior can be influenced by secondary goals imposed through instructions to the subject.

In conclusion, we have found that interception of fly balls by cancellation of the optical acceleration is only possible for a rather limited subset of ball trajectories. In addition, OA-based interception strategies other than the one investigated here will not be generally successful without taking into account other observable variables. What these variables are and how they are to be used to arrive at successful interception will be investigated in the near future.

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Appendix 1. Boundaries between finite and nonfinite catcher trajectories

The OAC solution given by Eq. 5 contains a term $(1+kt)^{-1}$, which may cause the solution to become nonfinite. This requires that k be sufficiently negative, causing the term $1+kt$ to become negative before the moment of interception. If t_i is an interception moment (i.e., $y_b = 0$ at $t = t_i$), then $D_i = 1 + kt_i$ will be called the finiteness determinant for that interception moment. If D_i is negative, the term $1+kt$ has passed zero between $t=0$ and $t=t_i$ and therefore the catcher's trajectory is nonfinite somewhere in this period. If D_i is positive, the trajectory is finite between $t=0$ and $t=t_i$.

Friction-dominated near-vertical ball trajectories

Friction-dominated near-vertical ball trajectories are described by a linear function of time: $y_b(t) = y_{b,0} + \dot{y}_{b,0}t$. It follows that there is a single interception possibility at $t_i = -y_{b,0}/\dot{y}_{b,0}$. The corresponding finiteness determinant equals:

$$\begin{aligned} D_i &= 1 + kt_i = 1 + \left(\frac{\dot{y}_{b,0}}{y_{b,0}} - \frac{\dot{x}_{bc,0}}{x_{bc,0}}\right)\left(-\frac{y_{b,0}}{\dot{y}_{b,0}}\right) \\ &= \frac{\dot{x}_{bc,0}}{x_{bc,0}}\frac{y_{b,0}}{\dot{y}_{b,0}} \end{aligned} \tag{8}$$

We consider here the case where the ball is before and above the catcher, falling down in the equilibrium between gravity and air friction ($x_{bc,0} > 0$, $y_{b,0} > 0$, $\dot{y}_{b,0} < 0$). Then the finiteness determinant is positive for $\dot{x}_{bc,0} < 0$. When the ball moves toward the catcher, his trajectory will be finite; if the ball moves away from him, it will be nonfinite. For $\dot{x}_{bc,0} = 0$ the finiteness determinant equals zero, and we must resort to the full solution. Substitution of $\dot{x}_{bc,0} = 0$ into this yields $x_{bc}(t) = 0$; the catcher remains still up to the moment the ball reaches the ground.

Friction-free ball trajectories

The situation without air friction is most conveniently analyzed in a global coordinate system with the catcher initially in rest at the origin ($x_{c,0} = 0, \dot{x}_{c,0} = 0$). The ball trajectory is described by $x_b(t)$ and $y_b(x_b(t))$, i.e., x_b is a function of time and y_b is a function of x_b. This yields the following expression for the coefficient k:

$$k = \frac{\dot{y}_{b,0}}{y_{b,0}} - \frac{\dot{x}_{b,0}}{x_{b,0}} = \left.\frac{\mathrm{d}y_b}{\mathrm{d}x_b}\right|_{x_{b,0}} \frac{\dot{x}_{b,0}}{y_{b,0}} - \frac{\dot{x}_{b,0}}{x_{b,0}} \tag{9}$$

For friction-free ball trajectories the horizontal ball velocity is constant; hence there is a simple relation between the intersection moment t_i and the horizontal intersection location $x_{b,i}$:

$$x_b = x_{b,0} + \dot{x}_{b,0}t \rightarrow t_i = \frac{x_{b,i} - x_{b,0}}{\dot{x}_{b,0}} \tag{10}$$

The value of the finiteness determinant then becomes:

$$D_i = 1 + kt_i = 1 + \left(\left.\frac{\mathrm{d}y_b}{\mathrm{d}x_b}\right|_{x_{b,0}} \frac{1}{y_{b,0}} - \frac{1}{x_{b,0}}\right)(x_{b,i} - x_{b,0}) \tag{11}$$

Interestingly, D_i depends only on the ball path, not explicitly on time. To evaluate D_i further, a general parabolic ball path is defined that passes through the initial position $[x_{b,0}\ y_{b,0}]^T$ and intersects the line of eye level at two arbitrary positions $x_{b,i1}$ and $x_{b,i2}$:

$$y_b = y_{b,0}\frac{(x_b - x_{b,i1})(x_b - x_{b,i2})}{(x_{b,0} - x_{b,i1})(x_{b,0} - x_{b,i2})} \tag{12}$$

Since $x_{b,i1}$ and $x_{b,i2}$ are mathematically indistinguishable, they may be ordered such that $x_{b,i1}$ occurs earlier in time than $x_{b,i2}$ (i.e., $t_{i1} < t_{i2}$). With this parametrization the determinant can be written as:

$$D_i = 1 + \left(\frac{1}{x_{b,0} - x_{b,i1}} + \frac{1}{x_{b,0} - x_{b,i2}} - \frac{1}{x_{b,0}}\right)(x_{b,i} - x_{b,0}) \tag{13}$$

By the definition of the ball path, $x_{b,i}$ is equal to either $x_{b,i1}$ or $x_{b,i2}$. The corresponding determinant values are:

$$D_{i1} = -\frac{(x_{b,0} - x_{b,i1})}{(x_{b,0} - x_{b,i2})}\frac{x_{b,i2}}{x_{b,0}} \quad D_{i2} = -\frac{(x_{b,0} - x_{b,i2})}{(x_{b,0} - x_{b,i1})}\frac{x_{b,i1}}{x_{b,0}} \tag{14}$$

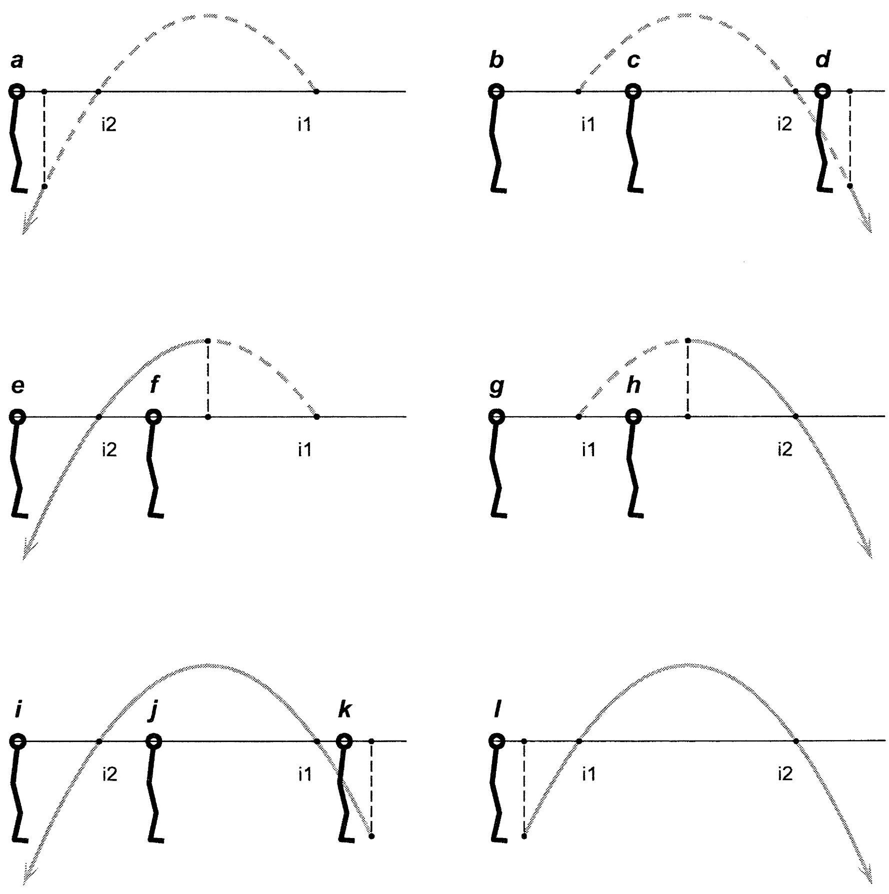

Fig. 8. The sign of the finiteness determinant D_i depends on the relative location of the ball-release position (beginning of drawn *arrow*), the interception points of the ball path with the line of eye level ($i1, i2$), and the initial position of the catcher (italic characters $a - l$). All possible orderings are shown in which the ball-release position is in front of the catcher's initial position. For further description see Appendix 1 and Table 1

Table 1. Possible combinations of the relative location of ball release position $x_{b,0}$, intersection points x_{b,i_1} and x_{b,i_2} and the initial position of the catcher ($x_{c,0} = 0$), with corresponding sign of the finiteness determinants D_{i1} and D_{i2}. Square brackets indicate that the finiteness determinant applies to an instant before $t = 0$ and therefore does not affect interception. The next to last column expresses the result in words; the final column gives the labels of the corresponding curves in Fig. 8

$t_{i1} < t_{i2} < 0$	$\dot{x}_{b,0} < 0$	$0 < x_{b,0} < x_{b,i2} < x_{b,i1}$	[D_{i1} < 0]	[D_{i2} < 0]	no interception	*a*
	$\dot{x}_{b,0} > 0$	$0 < x_{b,i1} < x_{b,i2} < x_{b,0}$	[D_{i1} < 0]	[D_{i2} < 0]	no interception	*b*
		$x_{b,i1} < 0 < x_{b,i2} < x_{b,0}$	[D_{i1} < 0]	[D_{i2} > 0]	no interception	*c*
		$x_{b,i1} < x_{b,i2} < 0 < x_{b,0}$	[D_{i1} > 0]	[D_{i2} > 0]	no interception	*d*
$t_{i1} < 0 < t_{i2}$	$\dot{x}_{b,0} < 0$	$0 < x_{b,i2} < x_{b,0} < x_{b,i1}$	[D_{i1} > 0]	D_{i2} > 0	finite	*e*
		$x_{b,i2} < 0 < x_{b,0} < x_{b,i1}$	[D_{i1} < 0]	D_{i2} > 0	finite	*f*
	$\dot{x}_{b,0} > 0$	$0 < x_{b,i1} < x_{b,0} < x_{b,i2}$	[D_{i1} > 0]	D_{i2} > 0	finite	*g*
		$x_{b,i1} < 0 < x_{b,0} < x_{b,i2}$	[D_{i1} > 0]	D_{i2} < 0	nonfinite	*h*
$0 < t_{i1} < t_{i2}$	$\dot{x}_{b,0} < 0$	$0 < x_{b,i2} < x_{b,i1} < x_{b,0}$	D_{i1} < 0	D_{i2} < 0	nonfinite	*i*
		$x_{b,i2} < 0 < x_{b,i1} < x_{b,0}$	D_{i1} > 0	D_{i2} < 0	finite up to t_{i1}	*j*
		$x_{b,i2} < x_{b,i1} < 0 < x_{b,0}$	D_{i1} > 0	D_{i2} > 0	finite	*k*
	$\dot{x}_{b,0} > 0$	$0 < x_{b,0} < x_{b,i1} < x_{b,i2}$	D_{i1} < 0	D_{i2} < 0	nonfinite	*l*

The signs of D_{i1} and D_{i2} are determined by the location of $x_{b,i1}, x_{b,i2}$, and $x_{b,0}$ relative to each other and to the origin (the catcher's initial position). We consider only initial ball positions the catcher can see when looking in the positive x-direction, i.e., $x_{b,0} > 0$. This leaves 12 possible orderings, which are shown in Fig. 8 and analyzed in Table 1. The results lead directly to the overview of Fig. 5.

Appendix 2. Boundaries between catcher trajectories with increasing and decreasing acceleration

For both friction-free and friction-dominated ball trajectories with x_b linear in time and y_b either parabolic or linear, the OAC solution (Eq. 5) may be expanded as:

$$x_c(t) = \frac{\alpha_{-1}}{1 + kt} + \alpha_0 + \alpha_1 t \tag{15}$$

where α_{-1}, α_1, and α_0 are constants (with $\alpha_1 = 0$ for the friction-dominated trajectories). Hence the second and third derivatives of x_c, are:

$$\ddot{x}_c = 2k^2 \frac{\alpha_{-1}}{(1+kt)^3}\ , \quad \dddot{x}_c = -6k^3 \frac{\alpha_{-1}}{(1+kt)^4} \tag{16}$$

It follows that the sign of $\ddot{x}_c$ does not change for catcher trajectories that are finite. Furthermore, the ratio $\dddot{x}_c/\ddot{x}_c$ is constant and equals $-3k$. Both facts combined allow the simple conclusion that for finite catcher trajectories the magnitude of the catcher's acceleration decreases with time if $k > 0$ and increases with time if $k < 0$. The boundary between these regimes is given by $k = \dot{y}_{b,0}/y_{b,0} - \dot{x}_{bc,0}/x_{bc,0} = 0$, which can be rewritten as:

$$\left.\frac{\mathrm{d}y_b}{\mathrm{d}x_{bc}}\right|_{t=0} = \frac{y_{b,0}}{x_{bc,0}} \tag{17}$$

The determining factor is the direction in which the ball is thrown away in relation to the direction in which the ball is initially seen by the catcher.

Appendix 3. Alternative control scheme

As long as the catcher remains at rest in the origin without accelerating, $x_{bc}(t) = x_b(t)$ and $y_{bc}(t) = y_b(t)$. Substitution of a general ball trajectory (Eq. 10 and Eq. 12) into Eq. 2 then yields the following elegant expression for the OA in this situation:

$$\ddot{h}_0 = -g \frac{x_{b,i1} x_{b,i2}}{x_b^3(t)} \tag{18}$$

For the catcher at rest, the sign of the OA depends on the positions where the ball path crosses eye level (in front of him or behind him) but is independent of the vertical position, in particular of whether the ball is above or below eye level.

If we constrain ourselves to balls that can be seen by the catcher ($x_b(t) > 0$) and assume that the ball is to be caught when it is descending (i.e., at position $x_{b,i2}$), we have the following four combinations of observed OA and required motion direction:

Crossing of ball path with eye level		Resulting sign of optical acceleration	Required motion direction	Required sign of κ
$x_{b,i1} < 0$	$x_{b,i2} < 0$	$-$	$-$ (backward)	$-$
$x_{b,i1} < 0$	$x_{b,i2} > 0$	$+$	$+$ (forward)	$-$
$x_{b,i1} > 0$	$x_{b,i2} < 0$	$+$	$-$ (backward)	$+$
$x_{b,i1} > 0$	$x_{b,i2} > 0$	$-$	$+$ (forward)	$+$

The table shows that there is no fixed relation between the sign of the OA and the direction of the desired motion. Essentially this is a result of the fact that the required direction is determined by $x_{b,i2}$ alone, whereas the sign of the OA is determined by the product $x_{b,i1}x_{b,i2}$. Hence the required sign of κ is determined by the sign of $x_{b,i1}$. For ball-game-type trajectories $x_{b,i1} > 0$ and $\kappa > 0$.

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